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Two-loop test of the $\mathcal{N} = 6$ Chern-Simons theory S-matrix

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Abstract: Starting from the integrable two-loop spin-chain Hamiltonian describing the anomalous dimensions of scalar operators in the planar $\mathcal{N} = 6$ superconformal Chern-Simons theory of ABJM, we perform a direct coordinate Bethe ansatz computation of the corresponding two-loop S-matrix. The result matches with the weak-coupling limit of the scalar sector of the all-loop S-matrix which we have recently proposed. In particular, we confirm that the scattering of $\mathcal{A}$ and $\mathcal{B}$ particles is reflectionless. As a warm up, we first review the analogous computation of the one-loop S-matrix from the one-loop dilatation operator for the scalar sector of planar $\mathcal{N} = 4$ superconformal Yang-Mills theory, and compare the result with the all-loop $SU(2|2)^2$ S-matrix.

Keywords: AdS-CFT Correspondence, Bethe Ansatz, Exact S-Matrix, Gauge-gravity correspondence

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Contents

1 Introduction

Exact factorized S-matrices [1] play a key role in the understanding of integrable models. Planar four-dimensional $\mathcal{N} = 4$ superconformal Yang-Mills (YM) theory (and therefore, according to the AdS_5/CFT_4 correspondence [2], a certain type IIB superstring theory on $AdS_5 \times S^5$) is believed to be integrable (see [3]–[6] and references therein). A corresponding exact factorized S-matrix with $SU(2|2)^2$ symmetry has been proposed (see [7]–[14] and references therein), which leads [8, 15, 16] to the all-loop Bethe ansatz equations (BAEs) [17].

Aharony, Bergman, Jafferis and Maldacena (ABJM) [18] recently proposed an analogous AdS_4/CFT_3 correspondence relating planar three-dimensional $\mathcal{N} = 6$ superconformal Chern-Simons (CS) theory to type IIA superstring theory on $AdS_4 \times CP^3$. Minahan and Zarembo [19] subsequently found that the scalar sector of $\mathcal{N} = 6$ CS is integrable at the leading two-loop order, and proposed two-loop BAEs for the full theory (see also [20]). Moreover, evidence for classical integrability of the dual string sigma model (large-coupling limit) was discovered in [21–23]. On the basis of these results, and assuming integrability to all orders, Gromov and Vieira then conjectured all-loop BAEs [24].

Based on the symmetries and the spectrum of elementary excitations [19, 25, 26], we proposed an exact factorized AdS_4/CFT_3 S-matrix [28]. As a check, we verified that this

S-matrix leads to the all-loop BAEs in [24]. An unusual feature of this S-matrix is that the scattering of $\mathcal{A}$ and $\mathcal{B}$ particles is reflectionless. (A similar S-matrix which is not reflectionless is not consistent with the known two-loop BAEs [29].) For further related developments of the AdS_4/CFT_3 correspondence, see [30] and references therein.

Considerable guesswork has entered into the above-mentioned all-loop results. While there is substantial evidence for the all-loop BAEs and S-matrix in the well-studied AdS_5/CFT_4 case, the same cannot be said for the rapidly-evolving AdS_4/CFT_3 case.

In an effort to further check our proposed S-matrix, we perform here a direct coordinate Bethe ansatz computation of the two-loop S-matrix, starting from the integrable two-loop spin-chain Hamiltonian describing the anomalous dimensions of scalar operators in planar $\mathcal{N}=6$ CS [19]. The result matches with the weak-coupling limit of the scalar sector of our all-loop S-matrix [28]. In particular, we confirm that the scattering of $\mathcal{A}$ and $\mathcal{B}$ particles is reflectionless. As a warm up, we first review the analogous computation by Berenstein and Vázquez [5] of the one-loop S-matrix from the one-loop dilatation operator for the scalar sector of planar $\mathcal{N}=4$ YM [3], and compare the result with the all-loop $\mathrm{SU}(2|2)^2$ S-matrix.

The outline of this paper is as follows. In section 2 we review the simpler case of $\mathcal{N}=4$ YM. In section 3 we analyze the $\mathcal{N}=6$ CS case, relegating most of the details of $\mathcal{A}-\mathcal{B}$ scattering to an appendix. We briefly discuss our results in section 4.

2 One-loop S-matrix in the scalar sector of $\mathcal{N}=4$ YM

As is well known, $\mathcal{N}=4$ YM has six scalar fields $\Phi_i(x)$ $(i=1,\ldots,6)$ in the adjoint representation of $\mathrm{SU}(N)$. It is convenient to associate single-trace gauge-invariant scalar operators with states of an $\mathrm{SO}(6)$ quantum spin chain with L sites,

$$\mathrm{tr}\,\Phi_{i_1}(x)\cdots\Phi_{i_L}(x) \quad \Leftrightarrow \quad |\Phi_{i_1}\cdots\Phi_{i_L}\rangle\,, \tag{2.1}$$

where Φ_i on the RHS are 6-dimensional elementary vectors with components $(\Phi_i)_j=\delta_{i,j}$. The one-loop anomalous dimensions of these operators are described by the integrable $\mathrm{SO}(6)$ quantum spin-chain Hamiltonian [3]

$$\Gamma=\frac{\lambda}{8\pi^2}H\,, \qquad H=\sum_{l=1}^{L}\left(1-\mathcal{P}_{l,l+1}+\frac{1}{2}K_{l,l+1}\right)\,, \tag{2.2}$$

where $\lambda=g_{YM}^2N$ is the 't Hooft coupling, $\mathcal{P}$ is the permutation operator,

$$\mathcal{P}\ \Phi_i\otimes\Phi_j=\Phi_j\otimes\Phi_i\,, \tag{2.3}$$

and the projector K acts as

$$K\ \Phi_i\otimes\Phi_j=\delta_{ij}\left(\sum_{k=1}^{6}\Phi_k\otimes\Phi_k\right)\,. \tag{2.4}$$

It is convenient to define the complex combinations

$$X=\Phi_1+i\Phi_2\,, \quad Y=\Phi_3+i\Phi_4\,, \quad Z=\Phi_5+i\Phi_6\,, \tag{2.5}$$

and to denote the corresponding complex conjugates with a bar, $\bar{X} = \Phi_1 - i\Phi_2$, etc. For $\phi_1 , \phi_2 \in \{X , \bar{X} , Y , \bar{Y} , Z , \bar{Z}\}$,

$$\mathcal{P}\ \phi_1 \otimes \phi_2 = \phi_2 \otimes \phi_1 \,, \tag{2.6}$$

and

$$K\ \phi_1 \otimes \phi_2 = \begin{cases} 0 & \text{if } \phi_1 \neq \bar{\phi}_2 \\ X \otimes \bar{X} + \bar{X} \otimes X + Y \otimes \bar{Y} + \bar{Y} \otimes Y + Z \otimes \bar{Z} + \bar{Z} \otimes Z & \text{if } \phi_1 = \bar{\phi}_2 \end{cases} . \tag{2.7}$$

2.1 Coordinate Bethe ansatz

We take $|Z^L\rangle$ as the vacuum state, which evidently is an eigenstate of H with zero energy. One-particle excited states ("magnons") with momentum p are given by

$$|\psi(p)\rangle_\phi = \sum_{x=1}^{L} e^{ipx} |x\rangle_\phi \,, \tag{2.8}$$

where

$$|x\rangle_\phi = |\ \overset{\overset{1}{\downarrow}}{Z} \cdots Z \overset{\overset{x}{\downarrow}}{\phi} Z \cdots \overset{\overset{L}{\downarrow}}{Z}\rangle \tag{2.9}$$

is the state obtained from the vacuum by replacing a single Z at site x with an "impurity" ϕ, which can be either $X , \bar{X} , Y, \bar{Y}$ (but not $\bar{Z}$, which can be regarded as a two-particle bound state). Indeed, one can easily check that (2.8) is an eigenstate of H with eigenvalue $E = \epsilon(p)$, where

$$\epsilon(p) = 4 \sin^2(p/2) \,. \tag{2.10}$$

In order to compute the two-particle S-matrix, we must construct all possible two-particle eigenstates. Let

$$|x_1, x_2\rangle_{\phi_1 \phi_2} = |\ \overset{\overset{1}{\downarrow}}{Z} \cdots \overset{\overset{x_1}{\downarrow}}{\phi_1} \cdots \overset{\overset{x_2}{\downarrow}}{\phi_2} \cdots \overset{\overset{L}{\downarrow}}{Z}\rangle \tag{2.11}$$

denote the state obtained from the vacuum by replacing the Z's at sites x_1 and x_2 with impurities ϕ_1 and ϕ_2, respectively, where $x_1 < x_2$. Following Berenstein and Vázquez [5], we distinguish the following three cases:

$\phi_1 = \phi_2$: the case of two particles of the same type (i.e., $\phi_1 = \phi_2 \equiv \phi \in \{X , \bar{X} , Y, \bar{Y}\}$) is equivalent to the well-known case originally considered by Bethe in his seminal investigation of the Heisenberg model. (See, e.g., the review by Plefka in [6].) The two-particle eigenstates are given by

$$|\psi\rangle = \sum_{x_1 < x_2} f(x_1, x_2)\, |x_1, x_2\rangle_{\phi\phi} \tag{2.12}$$

where

$$f(x_1, x_2) = e^{i(p_1 x_1 + p_2 x_2)} + S(p_2 \,, p_1)\, e^{i(p_2 x_1 + p_1 x_2)} \,. \tag{2.13}$$

Indeed, these states satisfy

$$H|\psi\rangle = E|\psi\rangle \tag{2.14}$$

with

$$E = \epsilon(p_1) + \epsilon(p_2)\,, \tag{2.15}$$

where $\epsilon(p)$ is given by (2.10). It also follows from (2.14) that the S-matrix for $\phi - \phi$ scattering is given by

$$S(p_2\,, p_1) = \frac{u_2 - u_1 + i}{u_2 - u_1 - i}\,, \tag{2.16}$$

where $u_j = u(p_j)$ and

$$u(p) = \frac{1}{2}\cot(p/2)\,. \tag{2.17}$$

$\phi_1 \ne \bar{\phi}_2$: if the two particles are not of the same type, but $\phi_1 \ne \bar{\phi}_2$, then the two-particle eigenstates are of the form

$$|\psi\rangle = \sum_{x_1<x_2} \left\{ f_{\phi_1\phi_2}(x_1, x_2)\, |x_1, x_2\rangle_{\phi_1\phi_2} + f_{\phi_2\phi_1}(x_1, x_2)\, |x_1, x_2\rangle_{\phi_2\phi_1} \right\}\,, \tag{2.18}$$

where

$$f_{\phi_i\phi_j}(x_1, x_2) = A_{\phi_i\phi_j}(12)\, e^{i(p_1x_1+p_2x_2)} + A_{\phi_i\phi_j}(21)\, e^{i(p_2x_1+p_1x_2)}\,. \tag{2.19}$$

One finds [5]

$$\begin{pmatrix} A_{\phi_1\phi_2}(21) \\ A_{\phi_2\phi_1}(21) \end{pmatrix} = \begin{pmatrix} R(p_2\,, p_1) & T(p_2\,, p_1) \\ T(p_2\,, p_1) & R(p_2\,, p_1) \end{pmatrix} \begin{pmatrix} A_{\phi_1\phi_2}(12) \\ A_{\phi_2\phi_1}(12) \end{pmatrix}\,, \tag{2.20}$$

where the transmission and reflection amplitudes are given by

$$T(p_2\,, p_1) = \frac{u_2 - u_1}{u_2 - u_1 - i}\,, \qquad R(p_2\,, p_1) = \frac{i}{u_2 - u_1 - i}\,, \tag{2.21}$$

respectively.

$\phi_1 = \bar{\phi}_2$: in the case $\phi_1 = \bar{\phi}_2 \in \{X\,, \bar{X}\,, Y, \bar{Y}\}$, the two-particle eigenstates are given by

$$\begin{aligned} |\psi\rangle = & \sum_{x_1<x_2} \sum_{\phi=X,Y} \left\{ f_{\phi\bar{\phi}}(x_1, x_2)\, |x_1, x_2\rangle_{\phi\bar{\phi}} + f_{\bar{\phi}\phi}(x_1, x_2)\, |x_1, x_2\rangle_{\bar{\phi}\phi} \right\} \\ & + \sum_{x_1} f_{\bar{Z}}(x_1) |x_1\rangle_{\bar{Z}}\,, \end{aligned} \tag{2.22}$$

where $f_{\phi_i\phi_j}(x_1, x_2)$ are again given by (2.19), and

$$f_{\bar{Z}}(x_1) = A_{\bar{Z}}\, e^{i(p_1+p_2)x_1}\,. \tag{2.23}$$

One finds [5]

$$\begin{pmatrix} A_{X\bar{X}}(21) \\ A_{\bar{X}X}(21) \\ A_{Y\bar{Y}}(21) \\ A_{\bar{Y}Y}(21) \end{pmatrix} = \begin{pmatrix} R(p_2\,,p_1) & T(p_2\,,p_1) & S(p_2\,,p_1) & S(p_2\,,p_1) \\ T(p_2\,,p_1) & R(p_2\,,p_1) & S(p_2\,,p_1) & S(p_2\,,p_1) \\ S(p_2\,,p_1) & S(p_2\,,p_1) & R(p_2\,,p_1) & T(p_2\,,p_1) \\ S(p_2\,,p_1) & S(p_2\,,p_1) & T(p_2\,,p_1) & R(p_2\,,p_1) \end{pmatrix} \begin{pmatrix} A_{X\bar{X}}(12) \\ A_{\bar{X}X}(12) \\ A_{Y\bar{Y}}(12) \\ A_{\bar{Y}Y}(12) \end{pmatrix}, \tag{2.24}$$

where

$$\begin{aligned} T(p_2\,,p_1) &= \frac{(u_2-u_1)^2}{(u_2-u_1-i)(u_2-u_1+i)}\,, \\ R(p_2\,,p_1) &= \frac{-1}{(u_2-u_1-i)(u_2-u_1+i)}\,, \\ S(p_2\,,p_1) &= \frac{-i(u_2-u_1)}{(u_2-u_1-i)(u_2-u_1+i)}\,. \end{aligned} \tag{2.25}$$

2.2 Comparison with the all-loop S-matrix

We now wish to compare the above scattering amplitudes with the weak-coupling limit of the all-loop SU(2|2)⊗SU(2|2) S-matrix [8]–[14]. This check has not (to our knowledge) been presented elsewhere, and will serve as a useful guide for the $\mathcal{N}=6$ CS case. It is convenient to express the latter in terms of two mutually commuting sets of Zamolodchikov-Faddeev operators $A_i^\dagger(p)\,,\tilde{A}_i^\dagger(p)$ $(i=1,\ldots,4)$,

$$\begin{aligned} A_i^\dagger(p_1)\,A_j^\dagger(p_2) &= \sum_{i',j'} S_0(p_1,p_2)\,\widehat{S}_{i\,j}^{i'\,j'}(p_1,p_2)\,A_{j'}^\dagger(p_2)\,A_{i'}^\dagger(p_1)\,, \\ \tilde{A}_i^\dagger(p_1)\,\tilde{A}_j^\dagger(p_2) &= \sum_{i',j'} S_0(p_1,p_2)\,\widehat{S}_{i\,j}^{i'\,j'}(p_1,p_2)\,\tilde{A}_{j'}^\dagger(p_2)\,\tilde{A}_{i'}^\dagger(p_1)\,, \\ A_i^\dagger(p_1)\,\tilde{A}_j^\dagger(p_2) &= \tilde{A}_j^\dagger(p_2)\,A_i^\dagger(p_1)\,. \end{aligned} \tag{2.26}$$

We identify the scalar one-particle states as follows,

$$\begin{aligned} X(p) &= A_1^\dagger(p)\,\tilde{A}_2^\dagger(p)\,, \quad \bar{X}(p) = A_2^\dagger(p)\,\tilde{A}_1^\dagger(p)\,, \\ Y(p) &= A_2^\dagger(p)\,\tilde{A}_2^\dagger(p)\,, \quad \bar{Y}(p) = A_1^\dagger(p)\,\tilde{A}_1^\dagger(p)\,. \end{aligned} \tag{2.27}$$

The only non-vanishing amplitudes in the scalar sector are

$$\widehat{S}_{a\,a}^{a\,a}(p_1,p_2) = A\,, \qquad \widehat{S}_{a\,b}^{a\,b}(p_1,p_2) = \frac{1}{2}(A-B)\,, \qquad \widehat{S}_{a\,b}^{b\,a}(p_1,p_2) = \frac{1}{2}(A+B)\,, \tag{2.28}$$

where $a\,,b\in\{1\,,2\}$ with $a\neq b$. Here

$$\begin{aligned} A &= \frac{x_2^- - x_1^+}{x_2^+ - x_1^-}\,, \\ B &= -\left[\frac{x_2^- - x_1^+}{x_2^+ - x_1^-} + 2\frac{(x_1^- - x_1^+)(x_2^- - x_2^+)(x_2^- + x_1^+)}{(x_1^- - x_2^+)(x_1^- x_2^- - x_1^+ x_2^+)}\right], \end{aligned} \tag{2.29}$$

where $x_i^\pm = x(p_i)^\pm$ with

$$\frac{x^+}{x^-} = e^{ip}\,, \qquad x^+ + \frac{1}{x^+} - x^- - \frac{1}{x^-} = \frac{i}{g}\,, \tag{2.30}$$

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and $g = \sqrt{\lambda}/(4\pi)$. Moreover, the scalar factor is given by

$$S_0(p_1\,,p_2)^2 = \frac{x_1^- - x_2^+}{x_1^+ - x_2^-}\frac{1 - \frac{1}{x_1^+ x_2^-}}{1 - \frac{1}{x_1^- x_2^+}}\sigma(p_1\,,p_2)^2\,, \tag{2.31}$$

where $\sigma(p_1\,,p_2)$ is the BES dressing factor [12, 14]. In the weak-coupling ($g \to 0$) limit,

$$x^\pm \to \frac{1}{g}\left(u \pm \frac{i}{2}\right)\,. \tag{2.32}$$

Therefore

$$A \to \frac{u_1 - u_2 + i}{u_1 - u_2 - i}\,, \qquad B \to -1\,, \tag{2.33}$$

and

$$S_0^2 \to \frac{u_1 - u_2 - i}{u_1 - u_2 + i}\,, \tag{2.34}$$

since $\sigma(p_1\,,p_2) \to 1$.

For two particles of the same type, the scattering amplitude is evidently given by

$$S(p_1, p_2) \equiv \left(S_0(p_1,p_2)\,\widehat{S}_{a\,a}^{a\,a}(p_1,p_2)\right)^2 = S_0^2\,A^2 \to \frac{u_1 - u_2 + i}{u_1 - u_2 - i}\,, \tag{2.35}$$

in agreement with (2.16).

We now consider the case $\phi_1 \ne \bar\phi_2$, e.g.,

$$X(p_1)\,Y(p_2) = T(p_1,p_2)\,Y(p_2)\,X(p_1) + R(p_1,p_2)\,X(p_2)\,Y(p_1)\,. \tag{2.36}$$

It follows from (2.26)–(2.28) and (2.33), (2.34) that

$$\begin{aligned} T(p_1,p_2) &= \frac{1}{2}S_0^2 A(A-B) \to \frac{u_1 - u_2}{u_1 - u_2 - i}\,,\\ R(p_1,p_2) &= \frac{1}{2}S_0^2 A(A+B) \to \frac{i}{u_1 - u_2 - i}\,, \end{aligned} \tag{2.37}$$

in agreement with (2.21).

Finally, we consider the case $\phi_1 = \bar\phi_2$, e.g.,

$$\begin{aligned} X(p_1)\,\bar X(p_2) &= T(p_1,p_2)\,\bar X(p_2)\,X(p_1) + R(p_1,p_2)\,X(p_2)\,\bar X(p_1)\\ &\quad + S(p_1,p_2)\,Y(p_2)\,\bar Y(p_1) + S(p_1,p_2)\,\bar Y(p_2)\,Y(p_1)\,. \end{aligned} \tag{2.38}$$

It follows from (2.26)–(2.28) and (2.33), (2.34) that

$$\begin{aligned} T(p_1,p_2) &= \frac{1}{4}S_0^2(A-B)^2 \to \frac{(u_1-u_2)^2}{(u_1-u_2-i)(u_1-u_2+i)}\,,\\ R(p_1,p_2) &= \frac{1}{4}S_0^2(A+B)^2 \to \frac{-1}{(u_1-u_2-i)(u_1-u_2+i)}\,,\\ S(p_1,p_2) &= \frac{1}{4}S_0^2(A-B)(A+B) \to \frac{i(u_1-u_2)}{(u_1-u_2-i)(u_1-u_2+i)}\,, \end{aligned} \tag{2.39}$$

in agreement with (2.25).[1]

In short, the all-loop AdS_5/CFT_4 S-matrix correctly reproduces the $\mathcal{N} = 4$ YM one-loop scalar-sector scattering amplitudes, as expected. In the next section, we perform a similar check of the AdS_4/CFT_3 S-matrix.

[1] There is a sign discrepancy in $S(p_1, p_2)$ which perhaps can be reconciled by a gauge transformation in (2.38), e.g., $Y \to -Y$ while leaving others unchanged.

3 Two-loop S-matrix in the scalar sector of $\mathcal{N} = 6$ CS

The $\mathcal{N} = 6$ CS theory [18] has a pair of scalar fields $A_i(x)$ $(i = 1, 2)$ in the bifundamental representation $(\mathbf{N}, \bar{\mathbf{N}})$ of the $\mathrm{SU}(N) \times \mathrm{SU}(N)$ gauge group, and another pair of scalar fields $B_i(x)$ $(i = 1, 2)$ in the conjugate representation $(\bar{\mathbf{N}}, \mathbf{N})$. These fields can be grouped into SU(4) multiplets $Y^A(x)$,

$$Y^A = (A_1\,, A_2\,, B_1^\dagger\,, B_2^\dagger)\,, \qquad Y_A^\dagger = (A_1^\dagger\,, A_2^\dagger\,, B_1\,, B_2)\,. \tag{3.1}$$

Following [19], we associate single-trace gauge-invariant scalar operators with states of an alternating SU(4) quantum spin chain with $2L$ sites,

$$\mathrm{tr}\, Y^{A_1}(x)\, Y_{B_1}^\dagger(x) \cdots Y^{A_L}(x)\, Y_{B_L}^\dagger(x) \quad \Leftrightarrow \quad |Y^{A_1}\, Y_{B_1}^\dagger \cdots Y^{A_L}\, Y_{B_L}^\dagger\rangle\,, \tag{3.2}$$

where Y^A on the RHS are 4-dimensional elementary vectors with components $(Y^A)_j = \delta_{A,j}$. The two-loop anomalous dimensions of these operators are described by the integrable alternating SU(4) quantum spin-chain Hamiltonian [19]

$$\Gamma = \lambda^2 H\,, \qquad H = \sum_{l=1}^{2L} \left(1 - \mathcal{P}_{l,l+2} + \frac{1}{2}\{K_{l,l+1}\,, \mathcal{P}_{l,l+2}\} \right)\,, \tag{3.3}$$

where $\lambda = N/k$ is the 't Hooft coupling,[2] $\mathcal{P}$ is the permutation operator, and the projector K acts as

$$K\ Y^A \otimes Y_B^\dagger = \delta_B^A \sum_{C=1}^{4} Y^C \otimes Y_C^\dagger\,, \qquad K\ Y_B^\dagger \otimes Y^A = \delta_B^A \sum_{C=1}^{4} Y_C^\dagger \otimes Y^C\,. \tag{3.4}$$

That is,

$$\begin{aligned}
K\ A_i \otimes A_j^\dagger &= K\ B_i^\dagger \otimes B_j = \delta_{ij} \sum_{k=1}^{2} \left(A_k \otimes A_k^\dagger + B_k^\dagger \otimes B_k \right)\,,\\
K\ A_i^\dagger \otimes A_j &= K\ B_i \otimes B_j^\dagger = \delta_{ij} \sum_{k=1}^{2} \left(A_k^\dagger \otimes A_k + B_k \otimes B_k^\dagger \right)\,,\\
K\ A_i \otimes B_j &= K\ B_i \otimes A_j = K\ A_i^\dagger \otimes B_j^\dagger = K\ B_i^\dagger \otimes A_j^\dagger = 0\,.
\end{aligned} \tag{3.5}$$

3.1 Coordinate Bethe ansatz

Following [25, 26], we take the state with L pairs of $(A_1 B_1)$, i.e.,

$$|(A_1 B_1)^L\rangle \tag{3.6}$$

as the vacuum state, which evidently is an eigenstate of H with zero energy. It is convenient to label the $(A_1 B_1)$ pairs by $x \in \{1, \ldots, L\}$. There are two types of one-particle excited states with momentum p, called "$\mathcal{A}$-particles" and "$\mathcal{B}$-particles." The former are given by

$$|\psi(p)\rangle_\phi^{\mathcal{A}} = \sum_{x=1}^{L} e^{ipx} |x\rangle_\phi^{\mathcal{A}}\,, \tag{3.7}$$

[2] The action has two SU(N) Chern-Simons terms with integer levels k and $-k$, respectively.

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where

$$|x\rangle_{\phi}^{\mathcal{A}} = |\stackrel{\stackrel{1}{\downarrow}}{(A_1B_1)} \cdots \stackrel{\stackrel{x}{\downarrow}}{(\phi B_1)} \cdots \stackrel{\stackrel{L}{\downarrow}}{(A_1B_1)}\rangle \tag{3.8}$$

is the state obtained from the vacuum by replacing the A_1 from pair x with an "impurity" ϕ, which can be either A_2 or $B_2^\dagger$ (but not $B_1^\dagger$, which can be regarded as a two-particle bound state). Similarly, the "$\mathcal{B}$-particles" are given by

$$|\psi(p)\rangle_{\phi}^{\mathcal{B}} = \sum_{x=1}^{L} e^{ipx} |x\rangle_{\phi}^{\mathcal{B}} , \tag{3.9}$$

where

$$|x\rangle_{\phi}^{\mathcal{B}} = |\stackrel{\stackrel{1}{\downarrow}}{(A_1B_1)} \cdots \stackrel{\stackrel{x}{\downarrow}}{(A_1\phi)} \cdots \stackrel{\stackrel{L}{\downarrow}}{(A_1B_1)}\rangle \tag{3.10}$$

is the state obtained from the vacuum by replacing the B_1 from pair x with an "impurity" ϕ, which can be either $A_2^\dagger$ or B_2 (but not $A_1^\dagger$, which can be regarded as a two-particle bound state). Indeed, both (3.7) and (3.9) are eigenstates of H with eigenvalue $E = \epsilon(p)$, where $\epsilon(p)$ is given by (2.10).

In order to compute the two-particle S-matrix, we must construct all possible two-particle eigenstates.

3.1.1 $\mathcal{A} - \mathcal{A}$ scattering

Let

$$|x_1, x_2\rangle_{\phi_1\phi_2}^{\mathcal{A}\mathcal{A}} = |\stackrel{\stackrel{1}{\downarrow}}{(A_1B_1)} \cdots \stackrel{\stackrel{x_1}{\downarrow}}{(\phi_1 B_1)} \cdots \stackrel{\stackrel{x_2}{\downarrow}}{(\phi_2 B_1)} \cdots \stackrel{\stackrel{L}{\downarrow}}{(A_1B_1)}\rangle \tag{3.11}$$

denote the state obtained from the vacuum by replacing the A_1's from pairs x_1 and x_2 with impurities ϕ_1 and ϕ_2, respectively, where $x_1 < x_2$ and $\phi_i \in \{A_2 \,, B_2^\dagger\}$. We distinguish two cases:

$\phi_1 = \phi_2$: the case of two $\mathcal{A}$-particles of the same type (i.e., $\phi_1 = \phi_2 \equiv \phi \in \{A_2 \,, B_2^\dagger\}$) is again the same as in the Heisenberg model. The two-particle eigenstates are given by

$$|\psi\rangle = \sum_{x_1<x_2} f(x_1, x_2)\, |x_1, x_2\rangle_{\phi\phi}^{\mathcal{A}\mathcal{A}} \tag{3.12}$$

where $f(x_1, x_2)$ is given by (2.13). These states have energy (2.15), and the S-matrix is again given by (2.16),

$$S(p_2 \,, p_1) = \frac{u_2 - u_1 + i}{u_2 - u_1 - i} \,. \tag{3.13}$$

$\phi_1 \neq \phi_2$: if the two $\mathcal{A}$-particles are not of the same type (e.g., $\phi_1 = A_2\,, \phi_2 = B_2^\dagger$), then the two-particle eigenstates are of the form

$$|\psi\rangle = \sum_{x_1<x_2} \left\{ f_{\phi_1\phi_2}(x_1,x_2)\,|x_1,x_2\rangle^{\mathcal{A}\,\mathcal{A}}_{\phi_1\phi_2} + f_{\phi_2\phi_1}(x_1,x_2)\,|x_1,x_2\rangle^{\mathcal{A}\,\mathcal{A}}_{\phi_2\phi_1} \right\}, \tag{3.14}$$

where $f_{\phi_i\phi_j}(x_1,x_2)$ is again given by (2.19). Since K on these states is zero, the S-matrix is again given by (2.21),

$$T(p_2\,,p_1) = \frac{u_2-u_1}{u_2-u_1-i}\,, \qquad R(p_2\,,p_1) = \frac{i}{u_2-u_1-i}\,. \tag{3.15}$$

3.1.2 $\mathcal{B}-\mathcal{B}$ scattering

Let

$$|x_1,x_2\rangle^{\mathcal{B}\,\mathcal{B}}_{\phi_1\phi_2} = |\,\overset{\overset{1}{\downarrow}}{(A_1B_1)}\cdots\overset{\overset{x_1}{\downarrow}}{(A_1\phi_1)}\cdots\overset{\overset{x_2}{\downarrow}}{(A_1\phi_2)}\cdots\overset{\overset{L}{\downarrow}}{(A_1B_1)}\rangle \tag{3.16}$$

denote the state obtained from the vacuum by replacing the B_1's from pairs x_1 and x_2 with impurities ϕ_1 and ϕ_2, respectively, where $x_1 < x_2$ and $\phi_i \in \{A_2^\dagger\,, B_2\}$. The eigenstates with two $\mathcal{B}$-particle are given by expressions similar to those with two $\mathcal{A}$-particles (namely, (3.12) and (3.14) with $|x_1,x_2\rangle^{\mathcal{A}\,\mathcal{A}}_{\phi_i\phi_j} \leftrightarrow |x_1,x_2\rangle^{\mathcal{B}\,\mathcal{B}}_{\phi_i\phi_j}$), and we obtain the same results (3.13), (3.15) for the scattering amplitudes .

3.1.3 $\mathcal{A}-\mathcal{B}$ scattering

In order to analyze $\mathcal{A}-\mathcal{B}$ scattering, we define the states

$$\begin{aligned}
|x_1,x_2\rangle^{\mathcal{A}\,\mathcal{B}}_{\phi_1\phi_2} &= |\,\overset{\overset{1}{\downarrow}}{(A_1B_1)}\cdots\overset{\overset{x_1}{\downarrow}}{(\phi_1B_1)}\cdots\overset{\overset{x_2}{\downarrow}}{(A_1\phi_2)}\cdots\overset{\overset{L}{\downarrow}}{(A_1B_1)}\rangle\,,\\
|x_1,x_2\rangle^{\mathcal{A}\,\mathcal{B}}_{\phi_2\phi_1} &= |\,\overset{\overset{1}{\downarrow}}{(A_1B_1)}\cdots\overset{\overset{x_1}{\downarrow}}{(A_1\phi_2)}\cdots\overset{\overset{x_2}{\downarrow}}{(\phi_1B_1)}\cdots\overset{\overset{L}{\downarrow}}{(A_1B_1)}\rangle\,,
\end{aligned} \tag{3.17}$$

where $x_1 < x_2$ and $\phi_1 \in \{A_2\,, B_2^\dagger\}$, $\phi_2 \in \{A_2^\dagger\,, B_2\}$. We distinguish two cases:

$\phi_1 \neq \phi_2^\dagger$: if $\phi_1 \neq \phi_2^\dagger$ (e.g., $\phi_1 = A_2\,, \phi_2 = B_2$), then K on the states (3.17) is zero. As noted in [19], we are left with two decoupled SU(2) chains on the even and odd sites. Hence, there is trivial scattering between $\mathcal{A}$ and $\mathcal{B}$ particles.

$\phi_1 = \phi_2^\dagger$: if $\phi_1 = \phi_2^\dagger$ (e.g., $\phi_1 = A_2\,, \phi_2 = A_2^\dagger$), then the eigenstates are given by

$$\begin{aligned}
|\psi\rangle = &\sum_{x_1<x_2}\ \sum_{\phi=A_2,B_2^\dagger} \left\{ f_{\phi\phi^\dagger}(x_1,x_2)\,|x_1,x_2\rangle^{\mathcal{A}\,\mathcal{B}}_{\phi\phi^\dagger} + f_{\phi^\dagger\phi}(x_1,x_2)\,|x_1,x_2\rangle^{\mathcal{A}\,\mathcal{B}}_{\phi^\dagger\phi} \right\}\\
&+\sum_{x_1}\sum_{k=1}^{2} \left\{ f_{A_kA_k^\dagger}(x_1)|x_1\rangle_{A_kA_k^\dagger} + f_{B_k^\dagger B_k}(x_1)|x_1\rangle_{B_k^\dagger B_k} \right\},
\end{aligned} \tag{3.18}$$

where

$$|x\rangle_{\phi_i\phi_j} = |\,\overset{\overset{1}{\downarrow}}{(A_1B_1)}\cdots\overset{\overset{x}{\downarrow}}{(\phi_i\phi_j)}\cdots\overset{\overset{L}{\downarrow}}{(A_1B_1)}\rangle \tag{3.19}$$

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is the state obtained from the vacuum by replacing the $(A_1 B_1)$ pair at x with $(\phi_i \phi_j)$. We assume that $f_{\phi_i\phi_j}(x_1, x_2)$ are again given by (2.19), and

$$f_{\phi_i\phi_j}(x_1) = A_{\phi_i\phi_j}\, e^{i(p_1+p_2)x_1} \,. \tag{3.20}$$

After a lengthy computation (see the appendix for further details), we find

$$\begin{pmatrix} A_{A_2 A_2^\dagger}(21) \\ A_{A_2^\dagger A_2}(21) \\ A_{B_2^\dagger B_2}(21) \\ A_{B_2 B_2^\dagger}(21) \end{pmatrix} = \begin{pmatrix} 0 & T(p_2\,,p_1) & 0 & S(p_2\,,p_1) \\ T(p_2\,,p_1) & 0 & S(p_2\,,p_1) & 0 \\ 0 & S(p_2\,,p_1) & 0 & T(p_2\,,p_1) \\ S(p_2\,,p_1) & 0 & T(p_2\,,p_1) & 0 \end{pmatrix} \begin{pmatrix} A_{A_2 A_2^\dagger}(12) \\ A_{A_2^\dagger A_2}(12) \\ A_{B_2^\dagger B_2}(12) \\ A_{B_2 B_2^\dagger}(12) \end{pmatrix} \tag{3.21}$$

where

$$T(p_2\,,p_1) = \frac{u_1 - u_2}{u_1 - u_2 - i}\,, \qquad S(p_2\,,p_1) = \frac{i}{u_1 - u_2 - i}\,. \tag{3.22}$$

Note that the scattering is reflectionless.

Similar results can be obtained for $\mathcal{B} - \mathcal{A}$ scattering.

3.2 Comparison with the all-loop S-matrix

We now wish to compare the above scattering amplitudes with the weak-coupling limit of the all-loop SU(2|2) S-matrix [28]. It is convenient to express the latter in terms of two sets of Zamolodchikov-Faddeev operators $\mathcal{A}_i^\dagger(p)$, $\mathcal{B}_i^\dagger(p)$ $(i = 1, \ldots, 4)$ corresponding to the $\mathcal{A}$, $\mathcal{B}$ particles, respectively,

$$\mathcal{A}_i^\dagger(p_1)\; \mathcal{A}_j^\dagger(p_2) = \sum_{i',j'} S_0(p_1,p_2)\, \widehat{S}_{i\,j}^{i'j'}(p_1,p_2)\; \mathcal{A}_{j'}^\dagger(p_2)\; \mathcal{A}_{i'}^\dagger(p_1)\,, \tag{3.23}$$

$$\mathcal{B}_i^\dagger(p_1)\; \mathcal{B}_j^\dagger(p_2) = \sum_{i',j'} S_0(p_1,p_2)\, \widehat{S}_{i\,j}^{i'j'}(p_1,p_2)\; \mathcal{B}_{j'}^\dagger(p_2)\; \mathcal{B}_{i'}^\dagger(p_1)\,, \tag{3.24}$$

$$\mathcal{A}_i^\dagger(p_1)\; \mathcal{B}_j^\dagger(p_2) = \sum_{i',j'} \tilde{S}_0(p_1,p_2)\, \widehat{S}_{i\,j}^{i'j'}(p_1,p_2)\; \mathcal{B}_{j'}^\dagger(p_2)\; \mathcal{A}_{i'}^\dagger(p_1)\,, \tag{3.25}$$

$$\mathcal{B}_i^\dagger(p_1)\; \mathcal{A}_j^\dagger(p_2) = \sum_{i',j'} \tilde{S}_0(p_1,p_2)\, \widehat{S}_{i\,j}^{i'j'}(p_1,p_2)\; \mathcal{A}_{j'}^\dagger(p_2)\; \mathcal{B}_{i'}^\dagger(p_1)\,. \tag{3.26}$$

The absence of $\mathcal{A}_{j'}^\dagger(p_2)\; \mathcal{B}_{i'}^\dagger(p_1)$ terms on the RHS of (3.25) (and similarly, of $\mathcal{B}_{j'}^\dagger(p_2)\; \mathcal{A}_{i'}^\dagger(p_1)$ terms on the RHS of (3.26)) means that the scattering is reflectionless.

We identify the scalar one-particle states as follows,

$$\begin{aligned} \mathcal{A}_1^\dagger(p)|0\rangle &= \sum_x e^{ipx}|x\rangle^{\mathcal{A}}_{A_2}\,, \qquad & \mathcal{A}_2^\dagger(p)|0\rangle &= \sum_x e^{ipx}|x\rangle^{\mathcal{A}}_{B_2^\dagger}\,, \\ \mathcal{B}_1^\dagger(p)|0\rangle &= \sum_x e^{ipx}|x\rangle^{\mathcal{B}}_{B_2}\,, \qquad & \mathcal{B}_2^\dagger(p)|0\rangle &= \sum_x e^{ipx}|x\rangle^{\mathcal{B}}_{A_2^\dagger}\,. \end{aligned} \tag{3.27}$$

The SU(2|2) S-matrix elements $\widehat{S}_{i\,j}^{i'j'}(p_1,p_2)$ are the same as before (2.28), (2.29), where $x^\pm$ satisfy (2.30) and [25–27]

$$g = h(\lambda)\,, \tag{3.28}$$

with $h(\lambda) \sim \lambda$ for small λ, and $h(\lambda) \sim \sqrt{\lambda/2}$ for large λ. The scalar factors are given by (cf. (2.31))

$$S_0(p_1\,,p_2) = \frac{1-\frac{1}{x_1^+ x_2^-}}{1-\frac{1}{x_1^- x_2^+}}\sigma(p_1\,,p_2)\,, \qquad \tilde{S}_0(p_1\,,p_2) = \frac{x_1^- - x_2^+}{x_1^+ - x_2^-}\sigma(p_1\,,p_2)\,. \tag{3.29}$$

In the weak-coupling ($g \to 0$) limit,

$$S_0 \to 1\,, \qquad \tilde{S}_0 \to \frac{u_1 - u_2 - i}{u_1 - u_2 + i}\,. \tag{3.30}$$

3.2.1 $\mathcal{A} - \mathcal{A}$ scattering

For two $\mathcal{A}$ particles of the same type (i.e., both $\mathcal{A}_a$ with $a \in \{1,2\}$), the scattering amplitude is evidently given by

$$S(p_1,p_2) \equiv S_0(p_1,p_2)\,\widehat{S}_{a\,a}^{a\,a}(p_1,p_2) = S_0\,A \to \frac{u_1 - u_2 + i}{u_1 - u_2 - i}\,, \tag{3.31}$$

in agreement with (3.13). Although the same expression also appears in the $\mathcal{N}=4$ YM case (2.35), note that the latter follows from the all-loop S-matrix (2.26) in a rather different way.

For two $\mathcal{A}$ particles of different type (i.e., $\mathcal{A}_a$ and $\mathcal{A}_b$ with $a, b \in \{1,2\}$ and $a \neq b$), it follows from (3.23), (2.28), (2.33) that

$$\mathcal{A}_a^\dagger(p_1)\;\mathcal{A}_b^\dagger(p_2) = T(p_1,p_2)\;\mathcal{A}_b^\dagger(p_2)\;\mathcal{A}_a^\dagger(p_1) + R(p_1,p_2)\;\mathcal{A}_a^\dagger(p_2)\;\mathcal{A}_b^\dagger(p_1)\,, \tag{3.32}$$

where

$$\begin{aligned} T(p_1,p_2) &= \frac{1}{2}S_0(A-B) \to \frac{u_1-u_2}{u_1-u_2-i}\,,\\ R(p_1,p_2) &= \frac{1}{2}S_0(A+B) \to \frac{i}{u_1-u_2-i}\,, \end{aligned} \tag{3.33}$$

in agreement with (3.15). Again, the same expressions arise in the $\mathcal{N}=4$ YM case (2.37) in a different way.

3.2.2 $\mathcal{B} - \mathcal{B}$ scattering

According to (3.24), the $\mathcal{B}-\mathcal{B}$ and $\mathcal{A}-\mathcal{A}$ scattering amplitudes are equal, in agreement with the results from section 3.1.2.

3.2.3 $\mathcal{A} - \mathcal{B}$ scattering

According to (3.25), the $\mathcal{A}_a - \mathcal{B}_a$ scattering amplitude is

$$\tilde{S}_0(p_1,p_2)\,\widehat{S}_{a\,a}^{a\,a}(p_1,p_2) = \tilde{S}_0\,A \to 1\,, \tag{3.34}$$

in agreement with the results from section 3.1.3 for the case $\phi_1 \neq \phi_2^\dagger$. Note that the scalar factor $\tilde{S}_0$ (3.29) is essential for obtaining this result.

For $\mathcal{A}_a - \mathcal{B}_b$ scattering (with $a, b \in \{1, 2\}$ and $a \neq b$), it follows from (3.25) that

$$\mathcal{A}_a^\dagger(p_1)\ \mathcal{B}_b^\dagger(p_2) = T(p_1, p_2)\ \mathcal{B}_b^\dagger(p_2)\ \mathcal{A}_a^\dagger(p_1) + S(p_1, p_2)\ \mathcal{B}_a^\dagger(p_2)\ \mathcal{A}_b^\dagger(p_1)\,, \tag{3.35}$$

where

$$\begin{aligned} T(p_1, p_2) &= \frac{1}{2}\tilde{S}_0(A - B) \to \frac{u_1 - u_2}{u_1 - u_2 + i}\,, \\ S(p_1, p_2) &= \frac{1}{2}\tilde{S}_0(A + B) \to \frac{i}{u_1 - u_2 + i}\,, \end{aligned} \tag{3.36}$$

which agrees with (3.22).[3]

4 Discussion

We have found that the all-loop AdS_4/CFT_3 S-matrix (3.23)–(3.26) correctly reproduces the $\mathcal{N} = 6$ CS two-loop scalar-sector scattering amplitudes. The scalar factors (3.29), which differ from the AdS_5/CFT_4 scalar factor (2.31), play a crucial role. In particular, we have confirmed that the scattering of $\mathcal{A}$ and $\mathcal{B}$ particles is reflectionless. This gives greater confidence in the correctness of the all-loop S-matrix, and in the corresponding all-loop BAEs [24].

We have restricted our analysis to the scalar sector of $\mathcal{N} = 6$ CS, since this is the only sector for which an explicit Hamiltonian has been available [19]. Very recently, the Hamiltonian for the full two-loop OSp(6|4) spin chain has been found [31, 32]. Hence, it should now be possible to extend the present analysis to other sectors, and thereby further check the all-loop S-matrix.

It would also be interesting to extend the present analysis beyond two loops. This could provide further information about the important function $h(\lambda)$ (3.28) and the dressing phase in the S-matrix. However, such an analysis must wait until the higher-loop Hamiltonian becomes available.

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[3]There is a sign discrepancy in $S(p_1, p_2)$. However, the sign of $S(p_1, p_2)$ in (3.35) can be changed by a gauge transformation, e.g. by changing $\mathcal{A}_1 \to -\mathcal{A}_1$ and leaving $\mathcal{A}_2\,, \mathcal{B}_1\,, \mathcal{B}_2$ unchanged.

A Details of $\mathcal{A}-\mathcal{B}$ scattering

In order to determine the $\mathcal{A}-\mathcal{B}$ scattering amplitudes, it is necessary to act with the Hamiltonian H (3.3) on the state (3.18). We catalog here the action of H on the various terms:

$$\begin{aligned} H|x_1\,,x_2\rangle^{\mathcal{A}\mathcal{B}}_{\phi_i,\phi_j} &= 4|x_1\,,x_2\rangle^{\mathcal{A}\mathcal{B}}_{\phi_i,\phi_j} - |x_1-1\,,x_2\rangle^{\mathcal{A}\mathcal{B}}_{\phi_i,\phi_j} - |x_1+1\,,x_2\rangle^{\mathcal{A}\mathcal{B}}_{\phi_i,\phi_j} \\ &\quad -|x_1\,,x_2-1\rangle^{\mathcal{A}\mathcal{B}}_{\phi_i,\phi_j} - |x_1\,,x_2+1\rangle^{\mathcal{A}\mathcal{B}}_{\phi_i,\phi_j} \quad \text{for } x_1 < x_2-1\,, \end{aligned} \tag{A.1}$$

$$\begin{aligned} H|x_1\,,x_1+1\rangle^{\mathcal{A}\mathcal{B}}_{A_2,A_2^\dagger} &= 4|x_1\,,x_1+1\rangle^{\mathcal{A}\mathcal{B}}_{A_2,A_2^\dagger} - |x_1\rangle_{A_2A_2^\dagger} - |x_1+1\rangle_{A_2A_2^\dagger} \\ &\quad -|x_1-1\,,x_1+1\rangle^{\mathcal{A}\mathcal{B}}_{A_2,A_2^\dagger} - |x_1\,,x_1+2\rangle^{\mathcal{A}\mathcal{B}}_{A_2,A_2^\dagger}\,, \end{aligned} \tag{A.2}$$

$$\begin{aligned} H|x_1\,,x_1+1\rangle^{\mathcal{A}\mathcal{B}}_{B_2^\dagger,B_2} &= 4|x_1\,,x_1+1\rangle^{\mathcal{A}\mathcal{B}}_{B_2^\dagger,B_2} - |x_1\rangle_{B_2^\dagger B_2} - |x_1+1\rangle_{B_2^\dagger B_2} \\ &\quad -|x_1-1\,,x_1+1\rangle^{\mathcal{A}\mathcal{B}}_{B_2^\dagger,B_2} - |x_1\,,x_1+2\rangle^{\mathcal{A}\mathcal{B}}_{B_2^\dagger,B_2}\,, \end{aligned} \tag{A.3}$$

$$\begin{aligned} H|x_1\,,x_1+1\rangle^{\mathcal{A}\mathcal{B}}_{A_2^\dagger,A_2} &= 4|x_1\,,x_1+1\rangle^{\mathcal{A}\mathcal{B}}_{A_2^\dagger,A_2} - \frac{1}{2}|x_1\rangle_{A_2^\dagger A_2} - \frac{1}{2}|x_1+1\rangle_{A_2A_2^\dagger} \\ &\quad +\frac{1}{2}|x_1\rangle_{A_1A_1^\dagger} + \frac{1}{2}|x_1+1\rangle_{A_1A_1^\dagger} + \frac{1}{2}|x_1\rangle_{B_1^\dagger B_1} + \frac{1}{2}|x_1+1\rangle_{B_1^\dagger B_1} \\ &\quad +\frac{1}{2}|x_1\rangle_{B_2^\dagger B_2} + \frac{1}{2}|x_1+1\rangle_{B_2^\dagger B_2} \\ &\quad -|x_1-1\,,x_1+1\rangle^{\mathcal{A}\mathcal{B}}_{A_2^\dagger,A_2} - |x_1\,,x_1+2\rangle^{\mathcal{A}\mathcal{B}}_{A_2^\dagger,A_2}\,, \end{aligned} \tag{A.4}$$

$$\begin{aligned} H|x_1\,,x_1+1\rangle^{\mathcal{A}\mathcal{B}}_{B_2,B_2^\dagger} &= 4|x_1\,,x_1+1\rangle^{\mathcal{A}\mathcal{B}}_{B_2,B_2^\dagger} - \frac{1}{2}|x_1\rangle_{B_2^\dagger B_2} - \frac{1}{2}|x_1+1\rangle_{B_2^\dagger B_2} \\ &\quad +\frac{1}{2}|x_1\rangle_{B_1^\dagger B_1} + \frac{1}{2}|x_1+1\rangle_{B_1^\dagger B_1} + \frac{1}{2}|x_1\rangle_{A_1A_1^\dagger} + \frac{1}{2}|x_1+1\rangle_{A_1A_1^\dagger} \\ &\quad +\frac{1}{2}|x_1\rangle_{A_2A_2^\dagger} + \frac{1}{2}|x_1+1\rangle_{A_2A_2^\dagger} \\ &\quad -|x_1-1\,,x_1+1\rangle^{\mathcal{A}\mathcal{B}}_{B_2,B_2^\dagger} - |x_1\,,x_1+2\rangle^{\mathcal{A}\mathcal{B}}_{B_2,B_2^\dagger}\,, \end{aligned} \tag{A.5}$$

$$\begin{aligned} H|x_1\rangle_{A_1A_1^\dagger} &= 3|x_1\rangle_{A_1A_1^\dagger} - \frac{1}{2}|x_1-1\rangle_{A_1A_1^\dagger} - \frac{1}{2}|x_1+1\rangle_{A_1A_1^\dagger} \\ &\quad +\frac{1}{2}|x_1\rangle_{A_2A_2^\dagger} + \frac{1}{2}|x_1+1\rangle_{A_2A_2^\dagger} + |x_1\rangle_{B_1^\dagger B_1} + |x_1+1\rangle_{B_1^\dagger B_1} \\ &\quad +\frac{1}{2}|x_1\rangle_{B_2^\dagger B_2} + \frac{1}{2}|x_1+1\rangle_{B_2^\dagger B_2} + \frac{1}{2}|x_1-1\,,x_1\rangle^{\mathcal{A}\mathcal{B}}_{A_2^\dagger,A_2} \\ &\quad +\frac{1}{2}|x_1\,,x_1+1\rangle^{\mathcal{A}\mathcal{B}}_{A_2^\dagger,A_2} + \frac{1}{2}|x_1-1\,,x_1\rangle^{\mathcal{A}\mathcal{B}}_{B_2,B_2^\dagger} + \frac{1}{2}|x_1\,,x_1+1\rangle^{\mathcal{A}\mathcal{B}}_{B_2,B_2^\dagger}\,, \end{aligned} \tag{A.6}$$

$$\begin{aligned} H|x_1\rangle_{A_2A_2^\dagger} &= 4|x_1\rangle_{A_2A_2^\dagger} + \frac{1}{2}|x_1-1\rangle_{A_1A_1^\dagger} + \frac{1}{2}|x_1\rangle_{A_1A_1^\dagger} \\ &\quad +\frac{1}{2}|x_1\rangle_{B_1^\dagger B_1} + \frac{1}{2}|x_1+1\rangle_{B_1^\dagger B_1} - |x_1-1\,,x_1\rangle^{\mathcal{A}\mathcal{B}}_{A_2,A_2^\dagger} - |x_1\,,x_1+1\rangle^{\mathcal{A}\mathcal{B}}_{A_2,A_2^\dagger} \\ &\quad -\frac{1}{2}|x_1-1\,,x_1\rangle^{\mathcal{A}\mathcal{B}}_{A_2^\dagger,A_2} - \frac{1}{2}|x_1\,,x_1+1\rangle^{\mathcal{A}\mathcal{B}}_{A_2^\dagger,A_2} \\ &\quad +\frac{1}{2}|x_1-1\,,x_1\rangle^{\mathcal{A}\mathcal{B}}_{B_2,B_2^\dagger} + \frac{1}{2}|x_1\,,x_1+1\rangle^{\mathcal{A}\mathcal{B}}_{B_2,B_2^\dagger}\,, \end{aligned} \tag{A.7}$$

$$
\begin{aligned}
H|x_1\rangle_{B_1^\dagger B_1} &= 3|x_1\rangle_{B_1^\dagger B_1} - \frac{1}{2}|x_1-1\rangle_{B_1^\dagger B_1} - \frac{1}{2}|x_1+1\rangle_{B_1^\dagger B_1} \\
&+\frac{1}{2}|x_1-1\rangle_{B_2^\dagger B_2} + \frac{1}{2}|x_1\rangle_{B_2^\dagger B_2} + |x_1-1\rangle_{A_1A_1^\dagger} + |x_1\rangle_{A_1A_1^\dagger} \\
&+\frac{1}{2}|x_1-1\rangle_{A_2A_2^\dagger} + \frac{1}{2}|x_1\rangle_{A_2A_2^\dagger} + \frac{1}{2}|x_1-1\,,x_1\rangle^{\mathcal{AB}}_{A_2^\dagger,A_2} \\
&+\frac{1}{2}|x_1\,,x_1+1\rangle^{\mathcal{AB}}_{A_2^\dagger,A_2} + \frac{1}{2}|x_1-1\,,x_1\rangle^{\mathcal{AB}}_{B_2,B_2^\dagger} + \frac{1}{2}|x_1\,,x_1+1\rangle^{\mathcal{AB}}_{B_2,B_2^\dagger}\,, \qquad \text{(A.8)} \\
H|x_1\rangle_{B_2^\dagger B_2} &= 4|x_1\rangle_{B_2^\dagger B_2} + \frac{1}{2}|x_1\rangle_{B_1^\dagger B_1} + \frac{1}{2}|x_1+1\rangle_{B_1^\dagger B_1} \\
&+\frac{1}{2}|x_1-1\rangle_{A_1A_1^\dagger} + \frac{1}{2}|x_1\rangle_{A_1A_1^\dagger} + \frac{1}{2}|x_1-1\,,x_1\rangle^{\mathcal{AB}}_{A_2^\dagger,A_2} \\
&+\frac{1}{2}|x_1\,,x_1+1\rangle^{\mathcal{AB}}_{A_2^\dagger,A_2} - |x_1-1\,,x_1\rangle^{\mathcal{AB}}_{B_2^\dagger,B_2} - |x_1\,,x_1+1\rangle^{\mathcal{AB}}_{B_2^\dagger,B_2} \\
&-\frac{1}{2}|x_1-1\,,x_1\rangle^{\mathcal{AB}}_{B_2,B_2^\dagger} - \frac{1}{2}|x_1\,,x_1+1\rangle^{\mathcal{AB}}_{B_2,B_2^\dagger}\,. \qquad \text{(A.9)}
\end{aligned}
$$

The appearance of terms of the form $|x\rangle_{A_kA_k^\dagger}$ and $|x\rangle_{B_k^\dagger B_k}$ ($k=1,2$) on the RHS of (A.2)–(A.5) explains the need for such terms in the eigenstate (3.18).

With the help of the above results, the eigenvalue equation

$$
H|\psi\rangle = E|\psi\rangle \qquad \text{(A.10)}
$$

with $|\psi\rangle$ and E given by (3.18) and (2.15), respectively, leads to the following equations for the amplitudes:

$$
\begin{aligned}
0 &= \left[3 - \frac{1}{2}(e^{i(p_1+p_2)} + e^{-i(p_1+p_2)}) - E\right] A_{A_1A_1^\dagger} \\
&+(1+e^{i(p_1+p_2)})\left(\frac{1}{2}A_{A_2A_2^\dagger} + A_{B_1^\dagger B_1} + \frac{1}{2}A_{B_2^\dagger B_2}\right) \\
&+\frac{1}{2}(e^{ip_2}+e^{-ip_1})\left[A_{A_2^\dagger A_2}(12) + A_{B_2B_2^\dagger}(12)\right] \\
&+\frac{1}{2}(e^{ip_1}+e^{-ip_2})\left[A_{A_2^\dagger A_2}(21) + A_{B_2B_2^\dagger}(21)\right]\,, \qquad \text{(A.11)} \\
0 &= \frac{1}{2}(1+e^{-i(p_1+p_2)})A_{A_1A_1^\dagger} + (4-E)A_{A_2A_2^\dagger} + \frac{1}{2}(1+e^{i(p_1+p_2)})A_{B_1^\dagger B_1} \\
&+(e^{ip_2}+e^{-ip_1})\left[-A_{A_2A_2^\dagger}(12) - \frac{1}{2}A_{A_2^\dagger A_2}(12) + \frac{1}{2}A_{B_2B_2^\dagger}(12)\right] \\
&+(e^{ip_1}+e^{-ip_2})\left[-A_{A_2A_2^\dagger}(21) - \frac{1}{2}A_{A_2^\dagger A_2}(21) + \frac{1}{2}A_{B_2B_2^\dagger}(21)\right]\,, \qquad \text{(A.12)} \\
0 &= (1+e^{-i(p_1+p_2)})\left[A_{A_1A_1^\dagger} + \frac{1}{2}A_{A_2A_2^\dagger} + \frac{1}{2}A_{B_2^\dagger B_2}\right] \\
&+\left[3 - \frac{1}{2}(e^{i(p_1+p_2)} + e^{-i(p_1+p_2)}) - E\right] A_{B_1^\dagger B_1} \\
&+\frac{1}{2}(e^{ip_2}+e^{-ip_1})\left[A_{A_2^\dagger A_2}(12) + A_{B_2B_2^\dagger}(12)\right] \\
&+\frac{1}{2}(e^{ip_1}+e^{-ip_2})\left[A_{A_2^\dagger A_2}(21) + A_{B_2B_2^\dagger}(21)\right]\,, \qquad \text{(A.13)}
\end{aligned}
$$

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$$
\begin{aligned}
0 &= \frac{1}{2}(1+e^{-i(p_1+p_2)})A_{A_1A_1^\dagger} + \frac{1}{2}(1+e^{i(p_1+p_2)})A_{B_1^\dagger B_1} + (4-E)A_{B_2^\dagger B_2} \\
&\quad +(e^{ip_2}+e^{-ip_1})\left[-A_{B_2^\dagger B_2}(12) + \frac{1}{2}A_{A_2^\dagger A_2}(12) - \frac{1}{2}A_{B_2B_2^\dagger}(12)\right] \\
&\quad +(e^{ip_1}+e^{-ip_2})\left[-A_{B_2^\dagger B_2}(21) + \frac{1}{2}A_{A_2^\dagger A_2}(21) - \frac{1}{2}A_{B_2B_2^\dagger}(21)\right], \qquad \text{(A.14)} \\
0 &= e^{ip_2}(4-e^{-ip_1}-e^{ip_2}-E)A_{A_2A_2^\dagger}(12) + e^{ip_1}(4-e^{-ip_2}-e^{ip_1}-E)A_{A_2A_2^\dagger}(21) \\
&\quad -(1+e^{i(p_1+p_2)})A_{A_2A_2^\dagger}, \qquad \text{(A.15)} \\
0 &= e^{ip_2}(4-e^{-ip_1}-e^{ip_2}-E)A_{B_2^\dagger B_2}(12) + e^{ip_1}(4-e^{-ip_2}-e^{ip_1}-E)A_{B_2^\dagger B_2}(21) \\
&\quad -(1+e^{i(p_1+p_2)})A_{B_2^\dagger B_2}, \qquad \text{(A.16)} \\
0 &= e^{ip_2}(4-e^{-ip_1}-e^{ip_2}-E)A_{A_2^\dagger A_2}(12) + e^{ip_1}(4-e^{-ip_2}-e^{ip_1}-E)A_{A_2^\dagger A_2}(21) \\
&\quad +\frac{1}{2}(1+e^{i(p_1+p_2)})\left(A_{A_1A_1^\dagger} - A_{A_2A_2^\dagger} + A_{B_1^\dagger B_1} + A_{B_2^\dagger B_2}\right), \qquad \text{(A.17)} \\
0 &= e^{ip_2}(4-e^{-ip_1}-e^{ip_2}-E)A_{B_2B_2^\dagger}(12) + e^{ip_1}(4-e^{-ip_2}-e^{ip_1}-E)A_{B_2B_2^\dagger}(21) \\
&\quad +\frac{1}{2}(1+e^{i(p_1+p_2)})\left(A_{A_1A_1^\dagger} + A_{A_2A_2^\dagger} + A_{B_1^\dagger B_1} - A_{B_2^\dagger B_2}\right). \qquad \text{(A.18)}
\end{aligned}
$$

Eliminating $A_{A_kA_k^\dagger}$, $A_{B_k^\dagger B_k}$ $(k=1,2)$, and then solving for the (21) amplitudes in terms of the (12) amplitudes, we arrive at the results (3.21), (3.22).

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